# Algebra2go: Working with Fractions and Mixed Numbers

Review of Fractions

1. Understand Fractions on a Number Line

Fractions are used to represent quantities between the whole numbers on a number line. A ruler or tape measure is basically a number line that has both whole numbers and fractions represented on their labels. When measuring objects with a ruler, an essential skill is to be able to read off measurements of length that involve mixed number representations such as $3\frac{1}{8}$ inches. To develop this skill, we will first look at some number lines with different fractional representations.
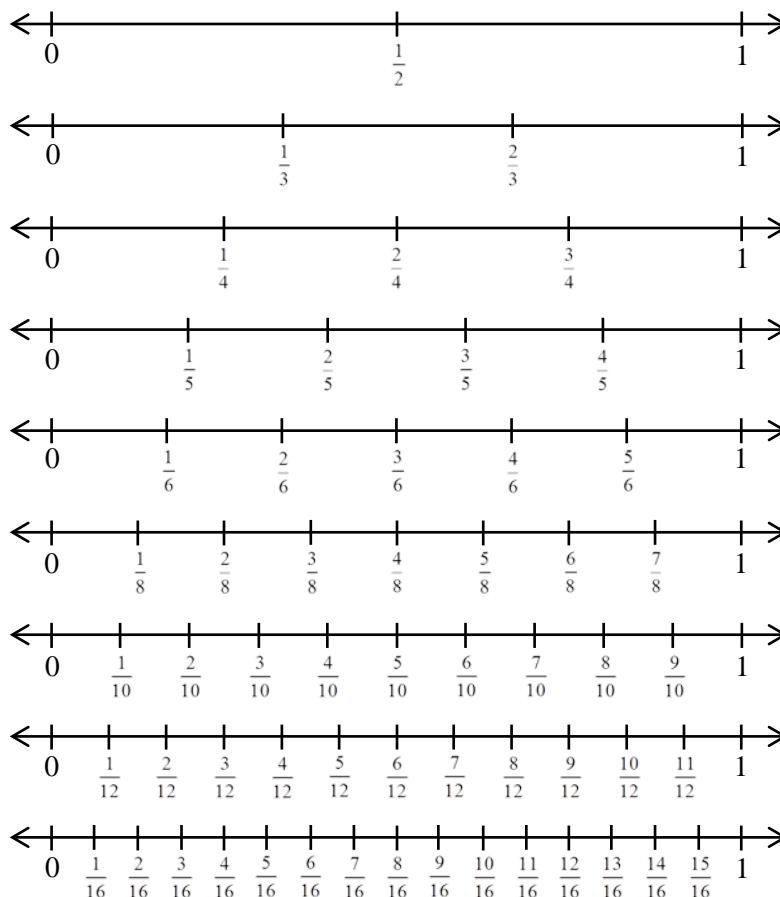

These number lines can be used to help us visually understand the meaning of equivalent fractions and the concept of basic mathematical operations with fractions. Additionally, they can help us master the skill of reading off measures using a variety of measuring scales.

Let's first discuss the meaning of equivalent fractions. Equivalent fractions have the same location on the number line. Since $\frac{3}{8}$ and $\frac{6}{16}$ have the same location on the number line, they are considered to be equivalent fractions. You may recall that the fraction $\frac{6}{16}$ can be reduced by dividing both the numerator and denominator by the common factor of 2. Doing so results in the fraction $\frac{3}{8}$.

Suppose you are asked to evaluate the expression $\frac{1}{3}+\frac{1}{4}+\frac{1}{6}$. You may recall, in order to add fractions, they must all be written with the same denominator. These fractions can each be re-written as an equivalent fraction with a denominator of 12.
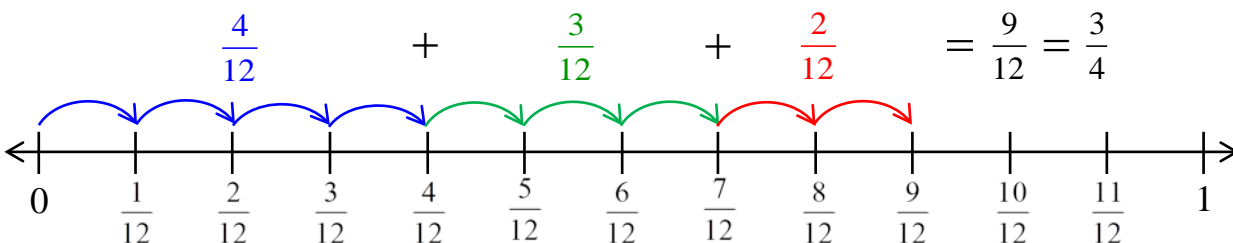
Using our fraction number lines, we can see that $\frac{1}{3}$ is equivalent to $\frac{4}{12}$, $\frac{1}{4}$ is equivalent to $\frac{3}{12}$, and $\frac{1}{6}$ is equivalent to $\frac{2}{12}$.
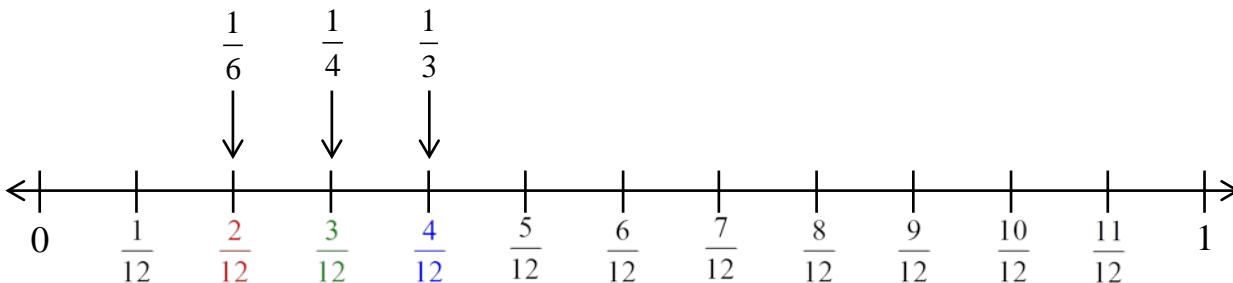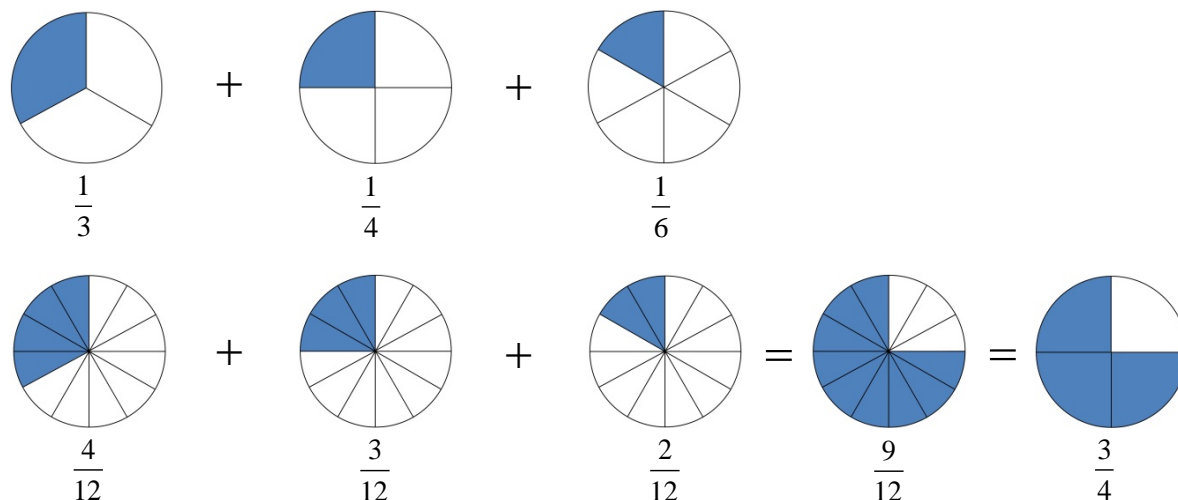

We can now evaluate the expression by replacing each fraction with its equivalent fraction.

$$\frac{1}{3}+\frac{1}{4}+\frac{1}{6}=\frac{4}{12}+\frac{3}{12}+\frac{2}{12}=\frac{4+3+2}{12}=\frac{9}{12}=\frac{3}{4}$$

This addition process can also be demonstrated on a number line as follows.

We can also use shaded areas to create a visual representation of this arithmetic operation.

$$\frac{1}{3} \qquad \frac{1}{4} \qquad \frac{1}{6}$$

$$\frac{4}{12} \qquad \frac{3}{12} \qquad \frac{2}{12} \qquad \frac{9}{12} \qquad \frac{3}{4}$$

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Use the fraction number line diagram to answer the following questions.

For Exercises 1 – 6, represent each fraction as an equivalent fraction with the indicated denominator.

1) $\dfrac{3}{4}$, 16

2) $\dfrac{1}{4}$, 12

3) $\dfrac{4}{10}$, 5

4) $\dfrac{12}{16}$, 4

5) $\dfrac{2}{3}$, 12

6) $\dfrac{1}{2}$, 10

7) Where is $\dfrac{0}{6}$ on the number line?

8) Where is $\dfrac{8}{8}$ on the number line?

For Exercises 9 – 12, use the fraction number line diagram to re-write each fraction with the same denominator. Then evaluate the expression.

9) $\dfrac{3}{5} - \dfrac{2}{10}$

10) $\dfrac{11}{16} - \dfrac{3}{8}$

11) $\dfrac{2}{3} - \dfrac{1}{2} + \dfrac{3}{4}$

12) $\dfrac{5}{6} - \dfrac{2}{3}$

13) Can $\dfrac{1}{3}$ be written as a fraction with a denominator of 16? Why or why not?

14) How do we represent the whole number 1 as a fraction with a denominator of 12?

2. Adding and Subtracting Unlike Fractions

Now, let's look at the arithmetic approach of finding the value of the expression $\dfrac{1}{3} + \dfrac{1}{4} + \dfrac{1}{6}$.

Recall that we need to make all the fractions have the same denominator before we add the fractions. Therefore, we first need to determine what the least common denominator (LCD) is.

Remember, the LCD is the smallest number that all the denominators divide evenly into.

The smallest number that a 3, 4, and 6 divide evenly into is 12. Therefore, 12 is our LCD. We now multiply each fraction by a factor of 1 to get the equivalent fraction having the LCD.

Remember, when multiplying fractions, multiply straight across the top, and straight across the bottom.

$$\frac{1}{3} = \frac{1}{3}(1) = \frac{1}{3}\left(\frac{4}{4}\right) = \frac{4}{12}$$

$$\frac{1}{4} = \frac{1}{4}(1) = \frac{1}{4}\left(\frac{3}{3}\right) = \frac{3}{12}$$

$$\frac{1}{6} = \frac{1}{6}(1) = \frac{1}{6}\left(\frac{2}{2}\right) = \frac{2}{12}$$

LCD = 12

$$\frac{1}{3} + \frac{1}{4} + \frac{1}{6}$$

$$\frac{4}{12} + \frac{3}{12} + \frac{2}{12}$$
Each fraction is now re-written with a denominator of 12.

$$\frac{4+3+2}{12}$$
Add the numerators of the fractions in the previous step. The denominator remains unchanged.

$$\frac{9}{12}$$
Reduce the fraction by dividing both the numerator and denominator by 3.

$$\boxed{\dfrac{3}{4}}$$

Example 1: Evaluate the expression $\dfrac{5}{8} - \dfrac{3}{4} + \dfrac{1}{2} - \dfrac{3}{16}$.

We first determine that the LCD = 16 since it is the smallest number that 8, 4, 2, and 16 divide evenly into. We now multiply each fraction by a factor of 1 to get its equivalent fraction having the LCD of 16.

$$\frac{5}{8} = \frac{5}{8}(1) = \frac{5}{8}\left(\frac{2}{2}\right) = \frac{10}{16}$$

$$\frac{3}{4} = \frac{3}{4}(1) = \frac{3}{4}\left(\frac{4}{4}\right) = \frac{12}{16}$$

$$\frac{1}{2} = \frac{1}{2}(1) = \frac{1}{2}\left(\frac{8}{8}\right) = \frac{8}{16}$$

$$\frac{3}{16} = \frac{3}{16}$$

LCD = 16

$$\frac{5}{8} - \frac{3}{4} + \frac{1}{2} - \frac{3}{16}$$

$$\frac{10}{16} - \frac{12}{16} + \frac{8}{16} - \frac{3}{16}$$ Each fraction is now re-written with a denominator of 16.

$$\frac{10 - 12 + 8 - 3}{16}$$ Add or subtract as indicated the numerators of the fractions in the previous step. The denominator remains unchanged.

$$\boxed{\frac{3}{16}}$$

For Exercises 15 – 22, identify the LCD and then re-write each fraction as an equivalent fraction having the LCD.

15) $\dfrac{2}{3}, \dfrac{5}{6}$ 16) $\dfrac{3}{4}, \dfrac{5}{8}$

17) $\dfrac{1}{9}, \dfrac{2}{3}$ 18) $\dfrac{2}{7}, \dfrac{2}{21}$

19) $\dfrac{5}{6}, \dfrac{1}{4}, \dfrac{3}{8}$ 20) $\dfrac{1}{2}, \dfrac{2}{3}, \dfrac{2}{5}$

21) $\dfrac{3}{8}, \dfrac{4}{5}, \dfrac{7}{10}$ 22) $\dfrac{9}{14}, \dfrac{3}{4}, \dfrac{6}{7}$

23) Where is the fraction $\dfrac{3}{2}$ on the number line?

24) Where is the fraction $\dfrac{7}{3}$ on the number line?

For Exercises 25 – 28, find the value of each expression.

25) $\dfrac{2}{3} - \dfrac{1}{6} + \dfrac{3}{2}$

26) $\dfrac{3}{4} - \dfrac{2}{3} + \dfrac{1}{6}$

27) $\dfrac{9}{7} - \dfrac{1}{2} - \dfrac{3}{4}$

28) $\dfrac{13}{8} - \dfrac{1}{5} - \dfrac{3}{4}$

3. Adding and Subtracting Mixed Numbers

Many calculations in Health Care careers require calculations that involve mixed numbers. A mixed number is simply the sum of a whole number and a fraction.

$$2\frac{2}{3} = 2 + \frac{2}{3} = \frac{6}{3} + \frac{2}{3} = \frac{6+2}{3} = \frac{8}{3}$$

Notice that we find that $2\frac{2}{3} = \frac{8}{3}$. Recall that the improper fraction $\frac{8}{3}$ represents a number that is greater than 1. We can also verify this by performing the division problem $8 \div 3$ which is represented by the fraction $\frac{8}{3}$.

$$\begin{array}{r} 2 \; R2 \\ 3\overline{)\,8} \\ \underline{-6} \\ 2 \end{array} \qquad 2\,R2 = 2\frac{2}{3} = \frac{8}{3}$$

Note: When we add mixed numbers, we can add the whole number parts and the fractional parts separately. Using these two results, we then write our answer in mixed number format.

Example 2: Find the value of $2\frac{2}{3} + 1\frac{1}{4}$ **. Write your answer in mixed number format.**

$$2\frac{2}{3} + 1\frac{1}{4} = \left(2 + 1\right) + \left(\frac{2}{3} + \frac{1}{4}\right) \qquad \text{Add the whole number parts and fractional parts separately.}$$

$$= \quad 3 \quad + \left(\frac{8}{12} + \frac{3}{12}\right) \qquad \text{Both fractions from the previous step are re-written as equivalent fractions with an LCD of 12.}$$

$$= \quad 3 \quad + \quad \left(\frac{11}{12}\right) \qquad \text{Add both fractions in the previous step.}$$

$$= \quad 3\frac{11}{12} \qquad \text{Write the final answer in mixed number format.}$$

When we subtract mixed numbers, we can subtract the whole number parts and the fractional parts separately. Using these two results, we again write our answer in mixed number format.

Note: These problems can be approached by first changing the mixed numbers into improper fractions and then performing the operation. This approach will be demonstrated later in this section.

Example 3: Find the value of $4\dfrac{5}{6} - 2\dfrac{2}{3}$ **. Write your answer in mixed number format.**

$$4\dfrac{5}{6} - 2\dfrac{2}{3} = \left(4 - 2\right) + \left(\dfrac{5}{6} - \dfrac{2}{3}\right)$$

Subtract the whole number parts and fractional parts separately.

$$= \quad 2 \quad + \left(\dfrac{5}{6} - \dfrac{4}{6}\right)$$

Notice that $\frac{2}{3}$ was re-written as an equivalent fraction with an LCD of 6.

$$= \quad 2 \quad + \left(\dfrac{1}{6}\right)$$

Subtract both fractions in the previous step.

$$= \quad 2\dfrac{1}{6}$$

Write the final answer in mixed number format.

Note: In some cases, when finding the sum of mixed numbers, we need to carry over a whole number from the fractional part. The following example demonstrates the process.

Example 4: Find the value of $2\dfrac{7}{8} + 1\dfrac{3}{4}$ **. Write your answer in mixed number format.**

$$2\dfrac{7}{8} + 1\dfrac{3}{4} = \left(2 + 1\right) + \left(\dfrac{7}{8} + \dfrac{3}{4}\right)$$

Add the whole number parts and fractional parts separately.

$$= \quad 3 \quad + \left(\dfrac{7}{8} + \dfrac{6}{8}\right)$$

Notice that $\frac{3}{4}$ was re-written as an equivalent fraction with an LCD of 8.

$$= \quad 3 \quad + \left(\dfrac{13}{8}\right)$$

To get this result, we added the fractional parts. Notice we have an improper fraction which will be re-written as a sum in the next step.

$$= \quad 3 \quad + \left(1 + \dfrac{5}{8}\right)$$

Here we write the improper fraction as a sum of a whole number and fraction.

$$= \quad 4 \quad + \left(\dfrac{5}{8}\right)$$

Here we added the whole numbers parts.

$$= \quad 4\dfrac{5}{8}$$

Write the final answer in mixed number format.

Note: In some cases, when finding the difference of mixed numbers, we need to borrow from the whole number part of the mixed number to avoid a negative result in the fractional part. The following example demonstrates the process.

Example 5: Find the value of $5\frac{2}{3} - 2\frac{3}{4}$. **Write your answer in mixed number format.**

$$5\frac{2}{3} - 2\frac{3}{4} = \left(5 - 2\right) + \left(\frac{2}{3} - \frac{3}{4}\right)$$

Subtract the whole number parts and fractional parts separately.

$$= \quad 3 \quad + \left(\frac{8}{12} - \frac{9}{12}\right)$$

From the previous step, both fractions are re-written as equivalent fractions with an LCD of 12. Notice that performing the subtraction operation would give us a negative result.

$$= \quad 2 \quad + \left(1 + \frac{8}{12} - \frac{9}{12}\right)$$

In this step, we borrowed a 1 from the whole number part to avoid the negative result.

$$= \quad 2 \quad + \left(\frac{12}{12} + \frac{8}{12} - \frac{9}{12}\right)$$

Here we represent the number 1 with an equivalent fraction having an LCD of 12.

$$= \quad 2 \quad + \quad \left(\frac{20}{12} - \frac{9}{12}\right)$$

Here we followed the rule for order of operations and worked left to right adding the first two fractions in the previous step.

$$= \quad 2 \quad + \quad \left(\frac{11}{12}\right)$$

Subtract both fractions in the previous step.

$$= \quad 2\frac{11}{12}$$

Write the final answer in mixed number format.

Example 6: Evaluate $4\frac{1}{10} + 2\frac{1}{2} - 3\frac{5}{6}$. **Write your answer in mixed number format.**

$$4\frac{1}{10} + 2\frac{1}{2} - 3\frac{5}{6} = \left(4 + 2 - 3\right) + \left(\frac{1}{10} + \frac{1}{2} - \frac{5}{6}\right)$$

As indicated, add and subtract the whole number parts and fractional parts separately.

$$= \quad 3 \quad + \left(\frac{3}{30} + \frac{15}{30} - \frac{25}{30}\right)$$

From the previous step, all fractions are re-written as equivalent fractions with an LCD of 30.

$$= \quad 3 \quad + \left(\frac{18}{30} - \frac{25}{30}\right)$$

Here we followed the rule for order of operations and worked left to right adding the first two fractions in the previous step. We can see that performing the subtraction in this step would give us a negative result.

$$= \quad 2 \quad + \left(1 + \frac{18}{30} - \frac{25}{30}\right)$$

Here we borrowed a 1 from the whole number part to avoid the negative result.

$$= \quad 2 \quad + \left(\frac{30}{30} + \frac{18}{30} - \frac{25}{30}\right)$$

Here we represent the number 1 with an equivalent fraction having an LCD of 30.

$$= \quad 2 \quad + \left(\frac{48}{30} - \frac{25}{30}\right)$$

Again we follow the rule for order of operations and work left to right adding the first two fractions in the previous step.

$$= \quad 2 \quad + \quad \left(\frac{23}{30}\right)$$

Subtract both fractions in the previous step.

$$= \quad 2\frac{23}{30}$$

Write the final answer in mixed number format.

As was mentioned earlier, we can first change the mixed numbers to improper fractions before we add or subtract. In some cases this is easier, and in some cases it may be more difficult. In Example 7, we will repeat the problem given in Example 5 using this approach.

Example 7: Find the value of $5\dfrac{2}{3} - 2\dfrac{3}{4}$ **by first writing the mixed numbers as improper fractions. Write your answer in mixed number format.**

$$5\dfrac{2}{3} - 2\dfrac{3}{4} = \dfrac{17}{3} - \dfrac{11}{4}$$ Both mixed numbers are re-written as improper fractions.

$$= \dfrac{68}{12} - \dfrac{33}{12}$$ From the previous step, both fractions are re-written as equivalent fractions with an LCD of 12.

$$= \dfrac{68-33}{12}$$ Here we write the difference of the two fractions in the previous step.

$$= \dfrac{35}{12}$$ Here we calculated the difference of 68 and 33. The denominator remains unchanged.

$$= 2R11$$ To get this result, we perform long division and divide 12 into 35.

$$= 2\dfrac{11}{12}$$ The result is finally written in mixed number format.

For Exercises 29 – 38, evaluate each expression.

29) $2\dfrac{1}{3} + 1\dfrac{1}{4}$ 30) $3\dfrac{7}{8} - 1\dfrac{3}{4}$

31) $3\dfrac{4}{5} + 1\dfrac{5}{6}$ 32) $4\dfrac{1}{3} - 2\dfrac{4}{7}$

33) $6\dfrac{7}{8} + 5\dfrac{7}{10}$ 34) $8\dfrac{3}{16} - 4\dfrac{5}{8}$

35) $7\dfrac{1}{2} + 6\dfrac{9}{10}$ 36) $11\dfrac{3}{4} - 5\dfrac{11}{16}$

37) $5\dfrac{4}{5} + 4\dfrac{5}{6} - 6\dfrac{2}{3}$

38) $2\dfrac{2}{3} - 1\dfrac{7}{8} + 9\dfrac{1}{4}$

For Exercises 39 – 44, evaluate each expression by first changing the mixed numbers to improper fractions. Write your final answer in mixed number format.

39) $3\dfrac{1}{10} + 2\dfrac{3}{8}$ 40) $4\dfrac{5}{6} - 2\dfrac{2}{3}$

41) $6\dfrac{3}{8} + 4\dfrac{1}{5}$ 42) $2\dfrac{1}{3} - 1\dfrac{1}{4}$

43) $6 - 3\dfrac{7}{10} + 2\dfrac{4}{5}$

44) $8\dfrac{3}{10} - 5 + 7$

Review Exercises

For Exercises 45 – 56, evaluate the expression.

45) $10 \cdot 9 \cdot 7$

46) $10 \cdot 6 \cdot 8$

47) $6 - 14 + 3$

48) $17 - 31 + 8$

49) $13 + 7 \cdot 6$

50) $11 - 2 \cdot 9$

51) $16 \div 4 \cdot 2$

52) $32 \div 2 \cdot 4$

53) $4 + 3^2 \cdot 2$

54) $40 - 20 \div 2^2$

55) $(11 - 7)^2 \div 2^3$

56) $(6 - 2)^3 - (2 \cdot 3)^2$

57) In your own words, write down the rules for order of operations.

58) When was the last time you had to perform a multiplication problem for a household task? Write down the event in your own words.

59) When was the last time you had to perform a division problem for a household task? Write down the event in your own words.

60) Besides math instructors, what type of careers use mathematics on a daily basis? Write down at least five careers.

0 $\dfrac{1}{2}$ 1

0 $\dfrac{1}{3}$ $\dfrac{2}{3}$ 1

0 $\dfrac{1}{4}$ $\dfrac{2}{4}$ $\dfrac{3}{4}$ 1

0 $\dfrac{1}{5}$ $\dfrac{2}{5}$ $\dfrac{3}{5}$ $\dfrac{4}{5}$ 1

0 $\dfrac{1}{6}$ $\dfrac{2}{6}$ $\dfrac{3}{6}$ $\dfrac{4}{6}$ $\dfrac{5}{6}$ 1

0 $\dfrac{1}{8}$ $\dfrac{2}{8}$ $\dfrac{3}{8}$ $\dfrac{4}{8}$ $\dfrac{5}{8}$ $\dfrac{6}{8}$ $\dfrac{7}{8}$ 1

0 $\dfrac{1}{10}$ $\dfrac{2}{10}$ $\dfrac{3}{10}$ $\dfrac{4}{10}$ $\dfrac{5}{10}$ $\dfrac{6}{10}$ $\dfrac{7}{10}$ $\dfrac{8}{10}$ $\dfrac{9}{10}$ 1

0 $\dfrac{1}{12}$ $\dfrac{2}{12}$ $\dfrac{3}{12}$ $\dfrac{4}{12}$ $\dfrac{5}{12}$ $\dfrac{6}{12}$ $\dfrac{7}{12}$ $\dfrac{8}{12}$ $\dfrac{9}{12}$ $\dfrac{10}{12}$ $\dfrac{11}{12}$ 1

0 $\dfrac{1}{16}$ $\dfrac{2}{16}$ $\dfrac{3}{16}$ $\dfrac{4}{16}$ $\dfrac{5}{16}$ $\dfrac{6}{16}$ $\dfrac{7}{16}$ $\dfrac{8}{16}$ $\dfrac{9}{16}$ $\dfrac{10}{16}$ $\dfrac{11}{16}$ $\dfrac{12}{16}$ $\dfrac{13}{16}$ $\dfrac{14}{16}$ $\dfrac{15}{16}$ 1